Math 131A-1: Homework 4

Due: April 22, 2015

1. Read Sections 10-11 in Ross.

2. Do problems 9.10, 9.12, 9.14, 10.6, 10.7, and 10.10 in Ross. [You may assume the result of 9.13 in writing up 9.14, but understanding this exercise on your own time is also recommended.]

3. We say a subset S of $\mathbb{R}$ is *closed* if whenever a sequence (s_n) of numbers in S converges to some s, the limit s is also in S.

   (a) Use exercise (8.9) from last week to show that that the interval $[a, b]$ is closed for any real $a < b$.

   (b) Give an example of a closed unbounded set in $\mathbb{R}$ (other than $\mathbb{R}$ itself).

   (c) Suppose $S \subset \mathbb{R}$ is closed and bounded above. Use exercise (10.7) above to show that S has a maximum.

4. Let (s_n) be the sequence $s_n = 1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^n}$.

   (a) Show that s_n is increasing and bounded above by 2. [Hint: Multiply s_n by 2^n, then use an induction formula for $2^n + 2^{n-1} + \cdots 2^2 + 2 + 1$ proved in the second lecture. Later in the course we will see a general formula for this sort of sum.]

   (b) Show that $s_{n+1} = \frac{1}{2}s_n + 1$, and conclude that $\lim s_n = 2$.

   (c) Let $t_n = 1 + 2 + 4 + \cdots 2^n$. Observe that $t_{n+1} = 2t_n + 1$. Why can't we conclude that $\lim t_n = -1$?

   Note that this addresses one of our motivational questions from the first lecture!

5. Don't forget that the first midterm is Wednesday April 20 during the usual class time. The exam will cover Sections 1-5, 7-10 in Ross. There's a sample midterm online, which you should look at to get an idea of the style and difficulty level of the test.